

# Different Number Systems

## 1 The Naturals

The natural numbers are the counting numbers.

They are 1, 2, 3, 4, 5, 6,... all the way to infinity.

Keep in mind that sometimes 0 is included as a natural number but not always.

An example of when we would use the natural numbers would be for counting people on the Earth.

We cannot have a negative amount of people. We also cannot have a fraction of a person. This is why we say that the set of all people on the Earth is defined on the natural numbers  $\mathbb{N}$ .

They are denoted as:  $\mathbb{N}$  or sometimes as just a boldface **N**.

$$\mathbb{N} = \{1, 2, 3, \dots\} \quad (1)$$

## 2 The Integers

The integers are the natural numbers  $\mathbb{N}$ , including 0, as well as all their 'negatives'.

They are ..., -3, -2, -1, 0, 1, 2, 3,... stretching from negative infinity to positive infinity.

Notice that the naturals are encompassed by the integers.

An example of the integers is best seen with a bank account. You can be in in the green or in the red; have money or owe money; positive or negative.

They are denoted as:  $\mathbb{Z}$  or sometimes as just a boldface **Z**.

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\} \quad (2)$$

## 3 The Rationals

The rational numbers are all numbers that can be expressed as the quotient of two integers  $\mathbb{Z}$ .

Choose two numbers a and b that are integers. Then  $\frac{a}{b}$  is a rational number. There are a few things to notice here. First, b can never be 0. Second, we very easily could choose our b to be 1 and then our rational number would remain an integer; whatever a was. Thus, integers (and by extension, the naturals) are encompassed by the rationals. Really all this means is that any and all fractions are rational numbers.

They are denoted as:  $\mathbb{Q}$  or sometimes as just a boldface **Q**.

## 4 The Irrationals

The irrational numbers are those numbers that cannot be expressed as a ratio of two integers. Examples are  $\sqrt{2}$  and the square roots of many other numbers, and special numbers like  $e$  and  $\pi$ . Irrational numbers have no exact decimal equivalents. To write any irrational number in decimal notation would require an infinite number of decimal digits.

$\bar{6}$ , for instance, is not an irrational number. Even though it has an infinite number of numbers after the decimal place, it can be expressed as the ratio of two integers. Namely, 2 and 3, as  $\frac{2}{3} = \bar{6}$ . If we start seeing patterns in the numbers following the decimal place that is a sign that the decimal can be expressed as a fraction.  $\sqrt{2} \approx 1.41421356237\dots$ . We could go on literally forever and never see a pattern or repetition in the decimal places. That's why we know  $\sqrt{2}$  is irrational.

## 5 The Reals

The real numbers are all the rational numbers and all the irrational numbers. In short, they are all numbers from negative infinity to positive infinity  $(-\infty, \infty)$ . They fill up a number line entirely. Think of number without an  $i$  next to it. That number is a real number.

They are denoted as:  $\mathbb{R}$  or just a boldface  $\mathbf{N}$ .

## 6 Imaginary Numbers

Imaginary numbers are the result of the square root of a negative number. They are denoted with a lowercase  $i$ . For example,  $\sqrt{-1} = i$  and  $\sqrt{-9} = 3i$ .

They are denoted as:  $\mathbb{C}$  or just a boldface  $\mathbf{C}$ .

So why does this all matter? Well, sometimes a problem will ask you to 'find all real solutions'. What does this mean? All the problem is asking you to do is find only answers without  $i$ 's in them.

**Example 6.1** *Are there any real solutions to the equation?*

$$x^2 + 51 = 26$$
$$x^2 = -25 \tag{3}$$

$$\sqrt{x^2} = \sqrt{-25} \tag{4}$$

$$x = \pm 5i \tag{5}$$

$$\tag{6}$$

*Because our answer has an  $i$  in it we would say that this equation has no real solutions.*