Free Body Diagrams

Problem Solving Outline and Example
Center for Academic Program Support

A. General Method for Solving Force Problems

1. Draw a separate free body diagram for each object of interest.
2. For each diagram, do the following:
   - Draw all forces on the free body as vectors coming out of the object.
   - Draw all other information (velocity, acceleration, etc.) away from the object.
   - Label an appropriate coordinate axis.
   - Identify and label the x- and y-components of all forces.
   - Write Newton's Second Law in component form:

$$\sum \vec{F} = m\vec{a} \;\Rightarrow\; \begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \end{cases}$$

   - Find $\sum F_x$ and $\sum F_y$ from the diagram.
   - Find ma_x and ma_y from your knowledge of the problem.
3. Write any other relevant equations, such as those for friction, centripetal acceleration, etc.
4. Solve for your answer

B. Some Things to Remember

1. Pay attention to sign conventions, based on your choice of axes.
2. Only draw forces on the object itself, not velocities or accelerations.
3. Only draw forces that act on the object.
4. Any object "at rest" or "at constant velocity" in any dimension (x or y) has that component of its acceleration (a_x or a_y) equal to zero.
5. Normal means "perpendicular to a surface." There is a normal force only if an object is against a surface, and it always points away from that surface.
6. Tension exists only if there is a rope/string pulling on an object. Tension always points along the rope/string, and it is the same at both ends. (This is important if you have a rope attached to two objects, as in a pulley problem.)
7. The centripetal force is not an extra force on the diagram. It is just a name for the net force in the center-pointing direction.

C. Example Problem

What angle of inclination of the ramp shown in FIG. (1) will cause a block of mass m (already in motion) to slide with constant velocity? The coefficient of kinetic friction between the block and the ramp is μ_k. How would your answer change for a block of mass $2m$?

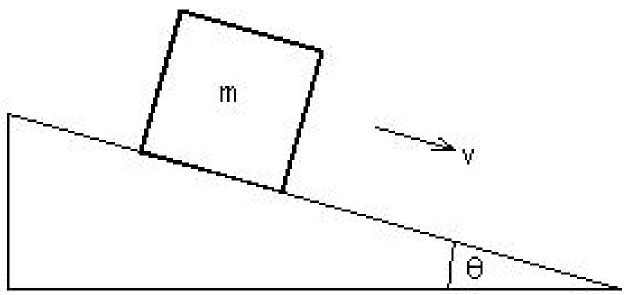

FIG. 1: Example problem

D. Solution

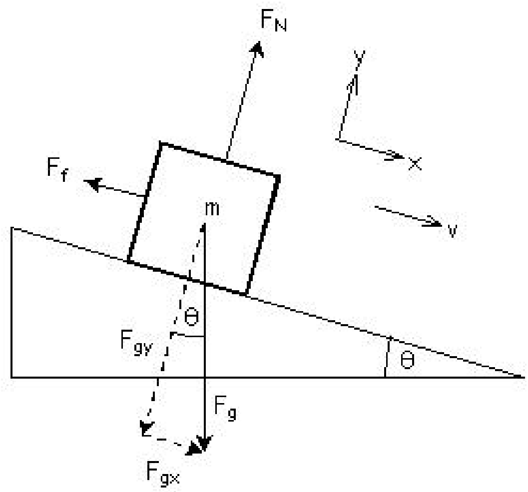

FIG. 2: Free Body Diagram

"Constant velocity" means that $a_x = 0$. Since the block is neither burrowing into the ramp, nor lifting off from it, $a_y = 0$ as well. The block has velocity v down the ramp therefore we must consider the *kinetic* friction force on the block defined as,

$$F_f = \mu_k F_N. \tag{1}$$

In this example the force of friction points only in the x-direction. The x and y components of the force of gravity on the block are,

$$F_{gx} = F_g \sin\theta \;=\; mg\sin\theta \tag{2a}$$
$$F_{gy} = F_g \cos\theta \;=\; mg\cos\theta \tag{2b}$$

Applying Newton's Second Law in the y-direction we find,

$$\begin{aligned} \sum F_y &= ma_y \\ F_N - F_{gy} &= 0 \qquad \text{(note the signs)} \\ F_N - mg\cos\theta &= 0 \qquad \text{(from Eqn (2b))} \\ F_N &= mg\cos\theta. \end{aligned} \tag{3}$$

The x component of Newton's Second Law gives,

$$\begin{aligned} \sum F_x &= ma_x \\ F_{gx} - F_f &= 0 \qquad \text{(note the signs)} \\ mg\sin\theta - \mu_k F_N &= 0 \qquad \text{(from Eqns (1) and (2a))} \\ mg\sin\theta &= \mu_k F_N. \end{aligned} \tag{4}$$

By substituting Eqn (3) into Eqn (4) we find,

$$\begin{aligned} \mu_k(mg\cos\theta) &= mg\sin\theta \\ \mu_k\cos\theta &= \sin\theta \\ \mu_k &= \tan\theta \\ \tan^{-1}(\mu_k) &= \theta. \end{aligned}$$

The angle required to allow the box to slide at a constant velocity is $\theta = \tan^{-1}(\mu_k)$, independent of the mass of the block.